

METHOD OF PSEUDOHOLOMORPHIC CURVES AND ITS APPLICATIONS

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Abstract. The method of "pseudoholomorphic" curves proved itself to be extremely useful in different fields. In symplectic topology ex. Gromov's non-squeezing theorem, Arnold conjecture and Floer homology, Gromov-Witten invariants. In complex analysis and geometry ex. polynomial hulls of totally real surfaces, envelopes of meromorphy, holomorphic foliations. We shall develop the theory of complex curves in almost complex manifolds and discuss some of these applications in our lectures.

Lecture 1. Complex Structures on the Plane and Beltrami Equation.

1.1. Varying Multiplication by an Imaginary Unit on the Plane. 1.2. J -holomorphic Functions and Beltrami Equation. 1.3. Sobolev Imbeddings and Cauchy-Green Operators. 1.4. Ein-satz of Vekua. 1.5. Holomorphic Motions. 1.6. Linear Algebra on a Complex Vector Space. 1.7. Almost Complex Manifolds. 1.8. Integrability. 1.9. Theorem of Nishino.

Lecture 2. Complex Curves in Almost Complex Manifolds.

2.1. Local Existence of J -Complex Curves. 2.2. Kobayashi-Royden Pseudometric. 2.3. Schwarz Lemmas and Complete Hyperbolicity. 2.4. J -Hermitian Metrics, Energy and Area. 2.5. Existence of Compatible and Tame Structures. 2.6. Symplectic Surfaces, First Chern Class and Genus Formula. 2.7. Primitivity and Positivity of Intersections. 2.8. Comparison Theorem. 2.9. Genus Formula for J -Complex Curves. 2.10. Optimal Regularity in Lipschitz Structures.

Lecture 3. Gromov Compactness Theorem.

3.1. A Priori Estimates and Convergence Outside of a Finite Set. 3.2. The Language of Parametrized Curves. 3.3. Compactness Theorem. 3.4. Totally Real Boundary Conditions and Reflection Principle. 3.5. Compactness Theorem for Curves With Boundary. 3.6. Attaching and Analytic Disc to a Lagrangian Submanifold. 3.7. Rational Convexity of Lagrangian Submanifolds.

Lecture 4. Moduli Spaces of J -Complex Curves.

4.1. Riemann-Roch Formula and Index of $\bar{\partial}$ -Type Operators. 4.2. Moduli Space of Rational Curves. 4.3. Universal Family and Evaluation Map. 4.4. Non-Squeezing Theorem. 4.5. Symplectic Capacities. 4.6. Gromov-Witten Invariant. 4.7. Quantum Multiplication and Quantum Cohomology. 4.8. Quantum Cohomology Ring of $\mathbb{C}P^n$. 4.9. Envelopes of Meromorphy and Continuity Principle. 4.10. Construction of Envelopes. 4.11. Examples.