

METHOD OF PSEUDOHOLOMORPHIC CURVES AND ITS APPLICATIONS

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Abstract. The method of "pseudoholomorphic" curves proved itself to be extremely useful in different fields. In symplectic topology ex. Gromov's non-squeezing theorem, Arnold conjecture and Floer homology, Gromov-Witten invariants. In complex analysis and geometry ex. polynomial hulls of totally real surfaces, envelopes of meromorphy, holomorphic foliations. We shall develop the theory of complex curves in almost complex manifolds and discuss some of these applications in our lectures.

Lecture 1. Complex Structures on the Plane and Beltrami Equation.

1.1. Varying Multiplication by an Imaginary Unit on the Plane. 1.2. J -holomorphic Functions and Beltrami Equation. 1.3. Sobolev Imbeddings and Cauchy-Green Operators. 1.4. Ein-satz of Vekua. 1.5. Holomorphic Motions. 1.6. Linear Algebra on a Complex Vector Space. 1.7. Almost Complex Manifolds. 1.8. Integrability. 1.9. Theorem of Nishino.

Lecture 2. Complex Curves in Almost Complex Manifolds.

2.1. Local Existence of J -Complex Curves. 2.2. Kobayashi-Royden Pseudometric. 2.3. Schwarz Lemmas and Complete Hyperbolicity. 2.4. J -Hermitian Metrics, Energy and Area. 2.5. Existence of Compatible and Tame Structures. 2.6. Symplectic Surfaces, First Chern Class and Genus Formula. 2.7. Primitivity and Positivity of Intersections. 2.8. Comparison Theorem. 2.9. Genus Formula for J -Complex Curves. 2.10. Optimal Regularity in Lipschitz Structures.

Lecture 3. Gromov Compactness Theorem.

3.1. A Priori Estimates and Convergence Outside of a Finite Set. 3.2. The Language of Parametrized Curves. 3.3. Compactness Theorem. 3.4. Totally Real Boundary Conditions and Reflection Principle. 3.5. Compactness Theorem for Curves With Boundary. 3.6. Attaching and Analytic Disc to a Lagrangian Submanifold. 3.7. Rational Convexity of Lagrangian Submanifolds.

Lecture 4. Moduli Spaces of J -Complex Curves.

4.1. Riemann-Roch Formula and Index of $\bar{\partial}$ -Type Operators. 4.2. Moduli Space of Rational Curves. 4.3. Universal Family and Evaluation Map. 4.4. Non-Squeezing Theorem. 4.5. Symplectic Capacities. 4.6. Gromov-Witten Invariant. 4.7. Quantum Multiplication and Quantum Cohomology. 4.8. Quantum Cohomology Ring of $\mathbb{C}\mathbb{P}^n$. 4.9. Envelopes of Meromorphy and Continuity Principle. 4.10. Construction of Envelopes. 4.11. Examples.

John Bland MONGE-AMPÈRE, EGUCHI-HANSON AND GRAVITONS AN OBSERVERS VIEWPOINT

Abstract: The singularity that lies at the centre of the Rossi example of a nonembeddable Cauchy Riemann structure also lies at the confluence of several streams of research, among them being nonlinear partial differential equations, invariant metrics, gravitons and a Penrose transform. In this series of talks, we will use the singularity of the Rossi example as our focal point to illustrate how these various streams come together and to suggest why Cauchy Riemann structures may provide a natural parameterization for the field. We will take the viewpoint of an outside observer to the field of thought, although at the end, we will indicate how an observer internal to the system might observe things. The talks will be essentially elementary in nature, although they will draw on background from a variety of sources.

Vladimir Ezhov (joint work with Martin Kolar, Gerd Schmalz) Normal forms and symmetries of real hypersurfaces of finite type

Abstract: We give a complete description of (not necessarily convergent) normal forms for real hypersurfaces of finite type in 2-dimensional complex space with respect to their holomorphic symmetry algebras. Our normal forms include refined versions of the constructions by Chern-Moser, Stanton, Kolar and Cartan. We use the method of simultaneous normalisation of the equations and the symmetries that goes back to Lie and Cartan. Our approach leads to a unique canonical equation of the hypersurface for every type of its symmetry algebra. We illustrate our results by explicitly normalising Cartan's homogeneous hypersurfaces and their automorphisms.

Alexander Isaev Extracting invariants of quasihomogeneous isolated hypersurface singularities from their moduli algebras.

Abstract: Let V be the germ of a complex hypersurface in \mathbb{C}^n at the origin having an isolated singularity, and $A(V)$ the Tjurina algebra (or moduli algebra) of V . By a theorem due to Mather and Yau, the algebra $A(V)$ and the dimension n determine V up to biholomorphic equivalence. Unfortunately, the proof of the Mather-Yau theorem does not provide an explicit procedure for recovering V from $A(V)$, and finding such a procedure is an interesting open problem. We discuss this problem in the light of our recent results in the quasihomogeneous case and explain how one can extract a number of invariants of V using classical invariant theory. Our procedure generalizes that proposed earlier by M. Eastwood.

Matthew Randall Local obstructions to 2-dimensional projective structures admitting skew-symmetric Ricci tensor

Abstract: A projective surface is a 2-dimensional manifold equipped with a projective structure i.e. a class of torsion-free affine connections that have the same geodesics as unparameterised curves. Given any projective surface we can ask whether it admits a torsion-free affine connection (in its projective class) that has skew-symmetric Ricci tensor. This is equivalent to solving a particular semi-linear overdetermined partial differential equation (PDE) that generalises the projective to Ricci-flat equation. It turns out that

there are local obstructions to solving the PDE in 2-dimensions. These obstructions are resultants of polynomials with coefficients constructed out of invariants of the projective structure.

Adam Harris Hyperbolicity and automorphisms of an unbounded domain

Abstract: The Kobayashi hyperbolicity and automorphisms of the unit ball in \mathbb{C}^2 are well known, and much is similarly known for bounded domains in general. By contrast, I will examine a special class of unbounded domains which have their origin in Eliashberg's theory of CW-decomposition of Stein manifolds. They are themselves Stein and hyperbolic, but have a much smaller automorphism group than the ball.