MATHEMATICS AND PHYSICS GRADUATION 2007
SWISS FEDERAL INSTITUTE OF TECHNOLOGY (ETH), ZURICH

CONFLICT AS CHALLENGE

BEATRICE BLEILE

Ladies and Gentlemen, Colleagues, Friends, Parents and Relatives,

Dear Graduands,

1. Congratulations

It is an honour and a pleasure to address you on this occasion.

Obtaining your Diploma is an achievement for which I heartily congratulate you.

This Diploma attests that you have mastered the challenges that physics and mathematics poses all students. The conferring of your Diploma marks the successful conclusion of your time as undergraduate students.

Take time to reflect and be proud of your work and your personal achievements in the course of your studies. Take time to look back on the proud history of ETH, which has been home to more than 21 Nobel Prize winners, and deservedly enjoys great international prestige.

The history of ETH imposes a responsibility on its graduates to maintain the proud tradition, irrespective of their future fields, as well as to engage themselves in the cause of science in general, and that of their alma mater in particular.

In view of the conflict between personal wishes and ambitions, and the positions available, one of the greatest challenges at the end of one's studies comprises the choice of a field to enter for the future, and the search for a position in this field. I shall return to the possibilities on offer, but first I would like to discuss conflicts, which have confronted me in my career.

2. Family vs Career

There is a conflict between family and career both for women and for men. While the manner in which this conflict presents itself depends on the individual women and men, it is also strongly dependent upon the social environment.

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I could find no-one to supervise me in a doctorate where we lived and where my partner had a tenured position. So, shortly after the birth of our son, I enrolled in a Masters programme in order to stay in touch with science. I worked on my Masters, and subsequently on my doctorate, only part time, spending a lot of time with our son.

I also spent a great deal of time teaching and am attempting only now, at the age of forty, and so at least ten years later than is generally the case, to establish an independent research career. I have been greeted with a great deal of support and understanding till now, but I am aware that this is not the case everywhere.

It is only natural that the matter of dividing time between family and career generate a certain measure of conflict. But I wish that we change our society in such a way that neither women nor men must choose between family and career but can both lead an enriching family life and pursue a fulfilling career.

3. Mathematics vs Physics

The different demands on mathematics and physics create a conflict, which is nowadays often viewed as an antagonism. Mathematics needs to be internally logically consistent and rigorous, but is free from external constraints. Physics, on the other hand, must describe the natural world. A physical theory, which does not describe physical reality, is useless, no matter how æsthetically satisfying it might be.

Numerous examples show that accepting this conflict as a challenge has enriched, and still does enrich, both disciplines: differential geometry and general relativity; group theory and quantum mechanics; super-symmetry, topological quantum field theory and Seiberg–Witten invariants for manifolds, ...

Today, I'd like to offer as an example the connection between robotics and topology.

4. Robotics und Topology

I am using the work of Michael Farber and seek the indulgence of experts in the audience, while also requesting them to correct me as needed.

The aim of robotics is to construct autonomous robots, capable of accepting complex instructions and of performing tasks without further human intervention. The instructions describe the tasks, and the robot decides for itself how these are to be carried out.

Topology enters robotics through the notion of a *configuration space*. The configuration space of a mechanical system is the space of all possible positions of the system. This is usually determined by a finite number of real parameters, and can therefore be regarded as a finite dimensional manifold.

Topology studies spaces and continuous functions between them, often with the help of invariants. For example, closed, oriented surfaces are determined up to homeomorphism by a single real number, the *Euler characteristic*.

To study general topological spaces, and distinguish them from each other, we not only use numerical invariants such as the Euler characteristic, but also introduce such algebraic structures as cohomology groups. These invariants can, in general, only distinguish between spaces up to continuous deformation.

Understanding the appropriate configuration space is enough to solve numerous problems in control theory. The topological properties of the configuration space can often explain the particularities of the behaviour of a system. I'd like to explain this a little more closely with the help of motion planning algorithms.

Let us write X for the configuration space of our mechanical system. A motion planning algorithm assigns to each pair of points, (A, B), in X a continuous path in X from the initial state A to the final state B. In order that this be always possible, there must be at least one such path between any pair of points in X. In other words, X must be *path connected.*

Now, a space Y is *simply connected* if every loop in Y can be contracted to a single point without leaving Y. Ordinary three dimensional space, in which we live and breath, is an example of a simply connected space. A torus, on the other hand, is not simply connected.

Let us consider the case when X is path connected. Our question is then whether there is a continuous motion planning algorithm, that is, an algorithm for which the path in X from A to B chosen depends continuously on A and B. It is not very difficult to show that there is a continuous motion planning algorithm if and only if X is simply connected.

That is why motion planning algorithms in practical applications are often not continuous, why investigating the instabilities and discontinuities is of practical importance. Farber introduced numerical invariants to describe the complexity of navigation within X.

One of these invariants is the *order of instability* of X. If the order of instability of X is r, then slight perturbations of the initial data can lead to up to r different motions generated by the motion planning algorithm.

The interplay between topology and robotics did not only pose interesting questions in topology, but also demonstrated to researchers in robotics that certain instabilities encountered in practice are characteristic of the configuration space and thus unavoidable.

For example, engineers attempted, unsuccessfully, to solve the problem of robot arms becoming stuck in certain positions by improving the mechanical construction or refining the motion planning algorithm. They finally turned to physicists and mathematicians for assistance, and it turned out that due to the properties of the configuration spaces in question, there are points at which the arm must get stuck. However, it was possible to find motion planning algorithms which avoid such points.

5. Integrity and Career

Mathematics and physics are hard masters, allowing no half measures, no cutting corners, thus instilling honesty and precision. Applying these self-same principles in other fields of working life can lead to conflict with colleagues or even superiors.

I know from personal experiences that opening one's mouth and speaking one's mind can have consequences, especially when doing so is not appreciated by one's superiors. But I can also report from personal experience that battles so arising can be won. It is, of course, sometimes necessary to make compromises, but it is important not to compromise one's integrity in the process.

6. Outlook

Dear Graduands, you face many challenges, but you also have many opportunities.

We need the next generation of researchers, for example, to understand the effect of climate change on our environment, and to be able to take appropriate action. But many other fields of research also need new blood, not the least basic research. An academic career is no easy path, but it is an extremely rewarding one. I encourage you to discuss prospects with your professors.

Teaching at a high school is also extremely important to our society. The education and training of the next generation is demanding, but also very rewarding. Teachers can open doors and have a life-long influence.

There are, of course, many other opportunities in industry, in banking and in insurance. For example, a few years ago, one of the sections of the Deutsche Bank in Sydney employed primarily people with doctorates in physics. Government agencies and services and many non-government organisations also need scientists.

What distinguishes physicists and mathematicians, and makes them so sought after, is that we do not shy away from difficult problems and are able to analyse them, to break them down into solvable parts in such a way that these partial solutions fit together to form a genuine solution of the total problem.

Whatever path you follow, your education and training at ETH prepared you well to make a significant contribution.

Thank you for your attention. I heartily congratulate you again on your diploma, and wish you much happiness and success in the future.

References

M. Farber, *Topology of robot motion planning*, in *Morse Theoretic Methods on Nonlinear Analysis and in Symplectic Topology*, P. Biran, O. Cornea, F. Lalonde (eds), NATO Science series, vol 217, 2006, 185–230.